

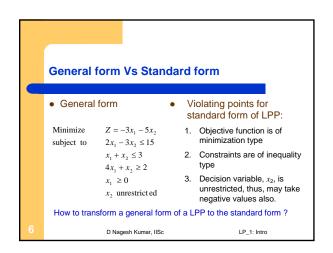
Standard form of LP problems

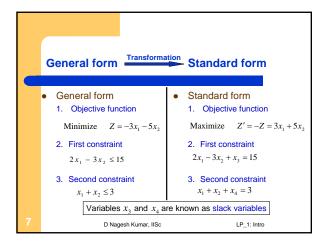
Standard form of LP problems must have following three characteristics:

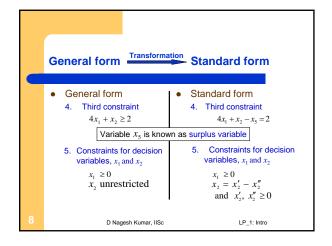
Discretive function should be of maximization type

All the constraints should be of equality type

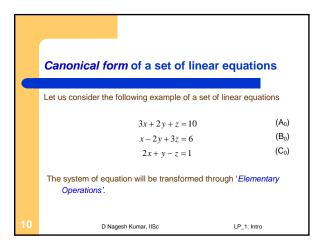
All the decision variables should be nonnegative







# Canonical form of LP Problems The 'objective function' and all the 'equality constraints' (standard form of LP problems) can be expressed in canonical form. Canonical form of LP problems is essential for simplex method (will be discussed later) Canonical form of a set of linear equations will be discussed next.



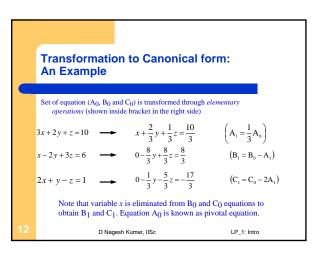
Elementary Operations

The following operations are known as elementary operations:

1. Any equation E<sub>r</sub> can be replaced by kE<sub>r</sub>, where k is a nonzero constant.

2. Any equation E<sub>r</sub> can be replaced by E<sub>r</sub> + kE<sub>s</sub>, where E<sub>s</sub> is another equation of the system and k is as defined above.

Note: Transformed set of equations through elementary operations is equivalent to the original set of equations. Thus, solution of transformed set of equations is the solution of original set of equations too.



### **Transformation to Canonical form:** Example contd.

Following similar procedure, y is eliminated from equation A<sub>1</sub> and C<sub>1</sub> considering B<sub>1</sub> as pivotal equation:

$$x+0+z=4 \qquad \left(A_2 = A_1 - \frac{2}{3}B_2\right)$$

$$0+y-z=-1 \qquad \left(B_2 = -\frac{3}{8}B_1\right)$$

$$0+0-2z=-6 \qquad \left(C_2 = C_1 + \frac{1}{2}B_2\right)$$

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### **Transformation to Canonical form:** Example contd.

Finally, z is eliminated form equation  $A_2$  and  $B_2$  considering  $C_2$  as pivotal equation :

$$x+0+0=1$$
  $(A_3 = A_2 - C_3)$   
 $0+y+0=2$   $(B_3 = B_2 + C_3)$   
 $0+0+z=3$   $(C_3 = -\frac{1}{2}C_2)$ 

Note: Pivotal equation is transformed first and using the transformed pivotal equation

The set of equations (A3, B3 and C3) is said to be in Canonical form which is equivalent to the original set of equations (A0, B0 and C0)

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### **Pivotal Operation**

Operation at each step to eliminate one variable at a time, from all equations except one, is known as pivotal

Number of *pivotal operations* are same as the number of variables in the set of equations.

Three pivotal operations were carried out to obtain the canonical form of set of equations in last example having three variables.

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**Transformation to Canonical form: Generalized procedure** 

Consider the following system of n equations with n variables

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
 (E<sub>1</sub>)

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$
  $(E_2)$   $\vdots$ 

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$
 (E<sub>n</sub>)

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### **Transformation to Canonical form:** Generalized procedure

Canonical form of above system of equations can be obtained by performing n pivotal operations

Variable  $x_i$  ( $i = 1 \cdots n$ ) is eliminated from all equations except j th equation for which  $a_{ii}$  is nonzero.

General procedure for one pivotal operation consists of following two steps,

- 1. Divide  $j^{\text{th}}$  equation by  $a_{ji}$ . Let us designate it as  $(E'_j)$ , i.e.,  $E'_j =$
- 2. Subtract  $a_{ki}$  times of  $(E'_i)$  equation from

k th equation  $(k = 1, 2, \dots, j-1, j+1, \dots, n)$ , i.e.,  $E_k - a_{ki}E'_j$ D Nagesh Kumar, IISc LP 1: Intro

### **Transformation to Canonical form: Generalized procedure**

After repeating above steps for all the variables in the system of equations, the canonical form will be obtained as follows:

$$1x_1 + 0x_2 + \dots + 0x_n = b_1''$$
 ( $E_1^c$ )

$$\begin{array}{lll} 0x_1 + 1x_2 + \cdots \cdots + 0x_n = b_2'' & (E_2') \\ \vdots & & \vdots \\ \vdots & & \vdots \\ 0x_1 + 0x_2 + \cdots \cdots + 1x_n = b_n'' & (E_n'') \end{array}$$

It is obvious that solution of above set of equation such as  $x_i = b_i^m$ is the solution of original set of equations also.

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## **Transformation to Canonical form: More general case**

Consider more general case for which the system of equations has m equation with n variables  $(n \ge m)$ 

$$\begin{array}{llll} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 & (E_1) \\ \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 & (E_2) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m & (E_m) \end{array}$$

It is possible to transform the set of equations to an equivalent canonical form from which at least one solution can be easily deduced

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## Transformation to Canonical form: More general case By performing n pivotal operations for any m variables (say, $x_1, x_2, \cdots x_m$ , called pivotal variables) the system of equations is reduced to canonical form as follows $1x_1 + 0x_2 + \cdots + 0x_m + a_{1,m+1}^n x_{m+1} + \cdots + a_{1,n}^n x_n = b_1^n \qquad (E_1^c)$ $0x_1 + 1x_2 + \cdots + 0x_m + a_{2,m+1}^n x_{m+1} + \cdots + a_{2,n}^n x_n = b_2^n \qquad (E_2^c)$ $\vdots$

 $(E_m^c)$ 

Variables,  $x_{m+1}, \cdots, x_n$ , of above set of equations is known as nonpivotal variables or independent variables.

 $0x_1 + 0x_2 + \dots + 1x_m + a''_{m,m+1}x_{m+1} + \dots + a''_{mn}x_n = b''_m$ 

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### Basic variable, Nonbasic variable, Basic solution, Basic feasible solution

One solution that can be obtained from the above set of equations is

$$x_i = b_i''$$
 for  $i = 1, \dots, m$   
 $x_i = 0$  for  $i = (m+1), \dots, n$ 

This solution is known as basic solution.

Pivotal variables,  $x_1, x_2, \dots x_m$ , are also known as *basic variables*.

Nonpivotal variables,  $x_{m+1}, \dots, x_n$ , are known as *nonbasic variables*.

Basic solution is also known as basic feasible solution because it satisfies all the constraints as well as non-negativity criterion for all the variables

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